

Pre-AP Geometry Date_____ 2.3 Notes
Deductive Reasoning (pp 90)

Equivalent Statements

Conditional Statement

$$p \rightarrow q$$

If the car will start,
then the battery is charged.

Contrapositive

$$\sim q \rightarrow \sim p$$

If the battery is not charged,
then the car will not start.

Equivalent Statements

Converse

$$q \rightarrow p$$

If the battery is charged,
then the car will start.

Inverse

$$\sim p \rightarrow \sim q$$

If the car will not start,
then the battery is not charged.

Law of Detachment

Definition	Characteristics
Example	Non-example

Law of Syllogism:

Definition	Characteristics
Example	Non-example

EXAMPLE 1***Using Symbolic Notation***

Let p be “the value of x is -5 ” and let q be “the absolute value of x is 5 .”

- Write $p \rightarrow q$ in words.
- Write $q \rightarrow p$ in words.
- Decide whether the biconditional statement $p \leftrightarrow q$ is true.

SOLUTION

- If the value of x is -5 , then the absolute value of x is 5 .
- If the absolute value of x is 5 , then the value of x is -5 .
- The conditional statement in part (a) is true, but its converse in part (b) is false. So, the biconditional statement $p \leftrightarrow q$ is false.

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Let p be “the value of x is ” and q be “the square of x is 16.”

1. Write $p \rightarrow q$ in words.
2. Write $q \rightarrow p$ in words.
3. Decide whether the biconditional statement $p \leftrightarrow q$ is true.

EXAMPLE 2 *Writing an Inverse and a Contrapositive*

Let p be “it is raining” and let q be “the soccer game is canceled.”

- a. Write the contrapositive of $p \rightarrow q$.
- b. Write the inverse of $p \rightarrow q$.

SOLUTION

a. **Contrapositive:** $\sim q \rightarrow \sim p$

If the soccer game is not canceled, then it is not raining.

b. **Inverse:** $\sim p \rightarrow \sim q$

If it is not raining, then the soccer game is not canceled.

Let p be “today is Monday” and q be “there is school.”

4. Write the contrapositive of $p \rightarrow q$.

5. Write the inverse of $p \rightarrow q$.

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Guided Practice: Let p be “a number is divisible by 3” and q be “a number is divisible by 6.”

6. Write $p \rightarrow q$ in words.

7. Write $q \rightarrow p$ in words.

8. Decide whether the biconditional statement $p \leftrightarrow q$ is true.

9. Write the inverse of $p \rightarrow q$.

EXAMPLE 3

Using Inductive and Deductive Reasoning

The following examples show how inductive and deductive reasoning differ.

- a. Andrea knows that Robin is a sophomore and Todd is a junior. All the other juniors that Andrea knows are older than Robin. Therefore, Andrea reasons *inductively* that Todd is older than Robin based on past observations.
- b. Andrea knows that Todd is older than Chan. She also knows that Chan is older than Robin. Andrea reasons *deductively* that Todd is older than Robin based on accepted statements.

Is the reasoning inductive or deductive?

10. Josh knows that Brand X computers cost less than Brand Y computers. All other brands that Josh knows of cost less than Brand X. Josh reasons that Brand Y costs more than all other brands.

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Is the reasoning inductive or deductive?

11. Josh knows that Brand X computers cost less than Brand Y computers. He also knows that Brand Y computers cost less than Brand Z. Josh reasons that Brand X costs less than Brand Z.

EXAMPLE 4

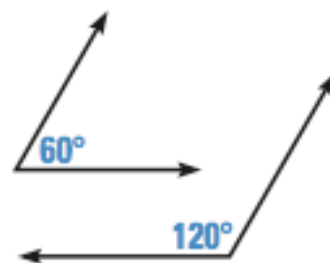
Using the Law of Detachment

State whether the argument is valid.

- Jamal knows that if he misses the practice the day before a game, then he will not be a starting player in the game. Jamal misses practice on Tuesday so he concludes that he will not be able to start in the game on Wednesday.
- If two angles form a linear pair, then they are supplementary;
 $\angle A$ and $\angle B$ are supplementary. So, $\angle A$ and $\angle B$ form a linear pair.

SOLUTION

- This logical argument is a valid use of the Law of Detachment. It is given that both a statement ($p \rightarrow q$) and its hypothesis (p) are true. So, it is valid for Jamal to conclude that the conclusion (q) is true.
- This logical argument is not a valid use of the Law of Detachment. Given that a statement ($p \rightarrow q$) and its conclusion (q) are true does not mean the hypothesis (p) is true. The argument implies that all supplementary angles form a linear pair.



The diagram shows that this is not a valid conclusion.



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State whether the argument is valid.

12. Michael knows that if he does not do his chores in the morning, he will not be allowed to play video games later the same day. Michael does not play video games on Friday afternoon. So Michael did not do his chores on Friday morning.

13. If two angles are vertical, then they are congruent. $\angle ABC$ and $\angle DBE$ are vertical. So $\angle ABC$ and $\angle DBE$ are congruent.

Guided Practice: State whether the argument is valid.

14. Sarah knows that all sophomores take driver education in her school. Hank takes driver education. So Hank is a sophomore.

15. If $m\angle ABC < 90^\circ$, then $\angle ABC$ is an acute angle. $m\angle ABC = 42^\circ$. So $\angle ABC$ is an acute angle.

FOCUS ON CAREERS



ZOOLOGY
Zoologists use facts about a bird's appearance to classify it. Ornithology is the branch of zoology that studies birds.

EXAMPLE 5

Using the Law of Syllogism

ZOOLOGY Write some conditional statements that can be made from the following true statements using the Law of Syllogism.

1. If a bird is the fastest bird on land, then it is the largest of all birds.
2. If a bird is the largest of all birds, then it is an ostrich.
3. If a bird is a bee hummingbird, then it is the smallest of all birds.
4. If a bird is the largest of all birds, then it is flightless.
5. If a bird is the smallest bird, then it has a nest the size of a walnut half-shell.

SOLUTION

Here are the conditional statements that use the Law of Syllogism.

- a. If a bird is the fastest bird on land, then it is an ostrich. (Use 1 and 2.)
- b. If a bird is a bee hummingbird, then it has a nest the size of a walnut half-shell. (Use 3 and 5.)
- c. If a bird is the fastest bird on land, then it is flightless. (Use 1 and 4.)

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Examples

Apply the law of syllogism to write a conditional statement.

1. If a fish swims at 68 mi/h, then it swims at 110 km/h.
2. If a fish can swim at 110 km/h, then it is a sailfish.
3. If a fish can not reach speeds to 68 m/hr, then it is not the fastest species of fish.

EXAMPLE 6

Using the Laws of Deductive Reasoning

Over the summer, Mike visited Alabama. Given the following true statements, can you conclude that Mike visited the Civil Rights Memorial?

If Mike visits Alabama, then he will spend a day in Montgomery.

If Mike spends a day in Montgomery, then he will visit the Civil Rights Memorial.

SOLUTION

Let p , q , and r represent the following.

p : Mike visits Alabama.

q : Mike spends a day in Montgomery.

r : Mike visits the Civil Rights Memorial.

Because $p \rightarrow q$ is true and $q \rightarrow r$ is true, you can apply the Law of Syllogism to conclude that $p \rightarrow r$ is true.

If Mike visits Alabama, then he will visit the Civil Rights Memorial.

- You are told that Mike visited Alabama, which means p is true. Using the Law of Detachment, you can conclude that he visited the Civil Rights Memorial.



**Civil Rights Memorial in
Montgomery, Alabama**

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16. Casey goes to a music store. Given the following true statements, can you conclude that Casey buys a CD? If Casey goes to a music store, she shops for a CD. If Casey shops for a CD, then Casey will buy a CD.

17. Write a conditional statement that can be made from the following true statements using the Law of Syllogism. If a plant is the largest plant on Earth, then the plant is a Sierra Redwood tree. If a plant is a Sierra Redwood tree, then the plant can weigh 1,800,000 kg.

18. If the statements $p \rightarrow q$ and $q \rightarrow r$ are true, then the statement

$p \rightarrow r$ is true by the Law of ____?____. If the statement $p \rightarrow q$ is true and p is true, then q is true by the Law of ____?____.

19. Given the notation for a conditional statement $p \rightarrow q$, what statement is represent by $q \rightarrow p$.

20. A conditional statement is defined by the notation $p \rightarrow q$. Write the symbolic notation for the inverse of $p \rightarrow q$.

21. If a ray bisects a right angle, then the congruent angles formed are complementary. In the diagram, $\angle ABC$ is a right angle. Are $\angle ABD$ & $\angle CBD$ complementary? Explain your reasoning.

